

## THE AGE OF THE OLDEST GLOBULAR CLUSTERS

M. SALARIS, S. DEGL'INNOCENTI, AND A. WEISS

Max-Planck-Institut für Astrophysik, Postfach 1523, 85740 Garching, Germany

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### ABSTRACT

The age of three of the oldest clusters—M15, M68, M92—has been redetermined. We use the latest equation of state (EOS) and opacity data available for calculating both isochrones and zero-age horizontal branches and employ the brightness difference between turnoff and horizontal branch to determine the cluster age. Our procedure and the uncertainties in obtaining absolute ages are discussed in detail. We find M68 to be  $12.2 \pm 1.8$  Gyr old; M15 appears to be either slightly younger or 2 Gyr older, depending on whether its age is determined differentially or directly. We discuss this discrepancy and argue that the lower age is within the real observational uncertainties. M92 is found to be slightly younger than M15 by about 0.5 Gyr. These ages are smaller by 1–2 Gyr compared to earlier work, and even smaller ages are possible. Our results help to reconcile cluster ages with recent results on the age of the universe determined from the Hubble constant.

*Subject headings:* cosmology: miscellaneous — globular clusters: general —  
globular clusters: individual (M15, M68, M92) — stars: evolution

### 1. INTRODUCTION

If the age of the universe is obtained from the cosmological expansion by determining the Hubble constant  $H_0$ , it turns out to be less than 15 Gyr for  $H_0 \gtrsim 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$  for values of the density parameter  $\Omega_0 \gtrsim 0.4$ . The most recent investigations into  $H_0$  (for a review, see van den Bergh 1994) yield values between 50 and 80, which limit the age of the universe to less than 15 Gyr for any reasonable  $\Omega_0$ , even if a cosmological constant is allowed. This number reduces to 10 Gyr, if  $H_0 \approx 75$  (from Cepheids or Type Ia supernovae). Clearly, even if one stretches the limits, the universe appears to be younger than  $\approx 15$  Gyr.

On the other hand, the classical method of determining the age of the oldest known stellar objects, the isochrone fitting to metal-poor globular clusters, consistently yields ages above  $\approx 14$  Gyr for the bulk of metal-poor halo clusters, with the majority of the clusters being around 16 Gyr and the oldest ones up to 18 Gyr old. An additional Gyr has to be added to this for the time between the genesis of the universe and the creation of the first clusters. This “conflict over the age of the universe” has been brought to a head by Bolte & Hogan (1995) with the example of M92, probably the best observed very old cluster, whose age they give as  $15.8 \pm 2.1$  Gyr (this range was used by Kennicutt, Freedman, & Mould 1995 to illustrate the relation—and conflict—with  $H_0$ ).

It therefore is necessary to reconsider age determinations of globular clusters and to investigate the errors more carefully. In this paper, we report new age determinations for three of the oldest clusters, including M92, using standard approaches but the latest equation of state and opacities for the stellar models.

Cluster ages were determined by using the difference in visual magnitude between main-sequence turnoff (TO) and horizontal branch (HB),  $\Delta(V)$ , the so-called vertical method. For a general discussion about various dating methods and the difficulties related to each of them, see, for example, Chaboyer, Sarajedini, & Demarque (1992) or Stetson, Vandenberg, & Bolte (1996). To yield reliable results, the  $\Delta(V)$ -method requires consistent photometry of both the turnoff region and the horizontal branch; the latter one should be

well populated to allow a precise definition of the luminosity and should have a large number of RR Lyrae variables, which are crucial for the distance determination. A review of the literature reveals that actually very few clusters fulfill these conditions; in our group of clusters, these are only M15 and M68. For many clusters,  $\Delta(V)$  has been determined by estimating the horizontal-branch brightness on the basis of very few or scattered stars. On the theoretical side, the isochrones used should consist of models of the appropriate composition, including the enhancement of  $\alpha$ -elements. The opacity and equation of state (EOS) data should reflect this. A calculation of horizontal-branch models with the same program is required as well.

Our work is similar to that of Chaboyer & Kim (1995, hereafter CK95), who used the same method and already found an average reduction of about 7% (or 1.2 Gyr for the oldest clusters) for the ages of 40 clusters owing to the use of equations of state including nonideal effects such as Coulomb-screening in the calculation of main-sequence and turnoff stars. However, they had determined  $\Delta(V)$  from the difference between the theoretical turnoff luminosity  $V_{\text{TO}}$  and an empirical horizontal-branch brightness  $V_{\text{HB}}$  (based on RR Lyrae luminosities). We emphasize that for several clusters (including M92 and M15), they had erroneously used published observational zero-age horizontal-branch (ZAHB) luminosities at the RR Lyrae instability strip, comparing them with an empirical relation for the mean (and not the ZAHB) RR Lyrae luminosity; this results in smaller  $\Delta(V)$ -values and ages (Chaboyer, Demarque, & Sarajedini 1996). Nevertheless, the effect of a systematic age reduction due to the improved EOS is real. In the present paper, we derive the absolute age of a cluster by comparing the observed  $\Delta(V)$  with calculated theoretical brightness differences. We therefore do not have the freedom to choose one of the many published  $M_v(\text{RR}) - [\text{Fe}/\text{H}]$  relations. Thus, our approach is completely self-consistent. Since we calculate theoretical ZAHBs as well and fit both isochrones and ZAHB to the observations, the influence of the new input physics is taken into account twice.

In the next section, we will describe all steps of our procedure to determine cluster ages in detail; § 3 contains the

age determination for the globular clusters M68, M15, and M92, where for the latter one we used a differential method. The conclusions will follow in the last section.

## 2. AGE DETERMINATION METHOD

### 2.1. Stellar Models

All the evolutionary calculations presented in this paper have been performed with the Frascati RAPHSON Newton Evolutionary Code (FRANEC) whose general features and physical inputs have already been described in previous papers (see, e.g., Chieffi & Straniero 1989). We have adopted the OPAL opacity tables (Rogers & Iglesias 1992; Iglesias & Rogers 1996; F. J. Rogers 1995, private communication; Rogers & Iglesias 1995) combined with the molecular opacities by Alexander & Ferguson (1994). More precisely, two sets of opacity tables (D. R. Alexander and C. A. Iglesias & F. J. Rogers, private communications) were used, one for a scaled solar metal mixture (Grevesse & Noels 1993) and one for the same heavy element fraction  $Z$ , but with the  $\alpha$ -elements being enhanced relative to the iron within the metal group. In particular, oxygen is enhanced by  $[O/Fe] = 0.5$ , and the other  $\alpha$ -elements by similar, but slightly varying, amounts, according to the observed values in low-metallicity stars (see, e.g., Wheeler, Sneden, & Truran 1989). It should be emphasized that the metal mixtures for both the low (molecular)- and high-temperature opacity tables are exactly the same. Thus, we were able to compare consistently stellar models with the same total metallicity but with different internal distributions of the heavy elements. In the high-density region, which is not covered by the OPAL opacities and in which the dominant source of opacity is due to electron conduction, we used the coefficients of Itoh et al. (1983).

As for the EOS, the updated OPAL EOS (Rogers 1994; Rogers, Swenson, & Iglesias 1996) has been used, upon which the OPAL opacities also rest. In regions where the OPAL EOS is not available, we supplemented it with the EOS described by Chieffi & Straniero (1989; for  $T < 5000$  K) and Straniero (1988; for degenerate He cores and the central regions of nondegenerate ZAHB He cores). We have verified that the transition from the OPAL to the supplementary EOS is smooth and without discontinuities.

Stellar models were evolved with a total metallicity of  $Z = 0.0002$  and  $Z = 0.0004$ , a scaled solar and an  $\alpha$ -enhanced metal distribution, and helium abundances  $Y = 0.23$  and  $Y = 0.24$ . These mixtures were chosen to match the observed values for the clusters considered in this investigation. For each given chemical composition, models with masses ranging from  $0.7$  to  $1 M_{\odot}$  were evolved from the zero-age main sequence (ZAMS) up to the red giant branch (RGB) to a luminosity of  $\log L/L_{\odot} \approx 1.7$ . We have chosen  $0.7 M_{\odot}$  as a lower mass limit for our models in order to have, at least for the evolution up to the turnoff point (TO), all the structure of our models covered by the OPAL EOS (see also the discussion in CK95). In all models, a mixing length of 1.6 (with the mixing-length formalism of Cox & Giuli 1968) was used; we have chosen this value in order to reproduce the observational data of Frogel, Persson, & Cohen (1983) for the temperatures of the RGBs of metal-poor clusters (following the procedure described by Chieffi, Straniero, & Salaris 1995). Isochrones were constructed by interpolating among evolutionary tracks for each selected chemical composition.

CK95 already have demonstrated the influence of the new OPAL EOS on the TO luminosity. In order to compare with their results and to show the influence on the ZAHB models, we have computed isochrones and ZAHB models with the Straniero EOS as well. Concerning the TO, the change to the OPAL EOS results in a decrease of the TO luminosity that depends both on metallicity and age. For  $Z \approx 10^{-4}$  it amounts to 0.03–0.1 mag (for ages of 16 and 12 Gyr). This compares well with CK95 (their Fig. 6, which, however, is for  $[Fe/H] = -1.3$ ) in the order of magnitude, although the range is slightly narrower (0.04 mag at 10 Gyr to 0.08 mag at 20 Gyr), and the relation between age and brightness decrease is the opposite.

The  $0.8 M_{\odot}$  models have been evolved up to the core helium flash to derive the mass of the helium core and the envelope He abundance after the first dredge-up; for this mass, the stellar age at the He flash is of the order of 13 Gyr, roughly representative of our derived age for metal-poor globular clusters (see the following section). As for the helium core masses at the helium flash ( $M_{He}$ ), we find quite a good agreement between the values obtained by adopting the OPAL and the Straniero EOS, for a fixed age.

We then constructed ZAHB models by employing the He core mass and the envelope chemical composition at the flash for the computation of a set of He-burning models with different masses of the H-rich envelopes. Following Castellani, Chieffi, & Pulone (1991), we took models already evolved by 1 Myr, which should represent quite accurately the theoretical counterparts of the lower luminosity boundaries of the observed HBs.

To investigate the influence of the new EOS on the ZAHB models, we computed theoretical ZAHB models for metallicities ranging from  $Z = 0.0001$  up to  $Z = 0.004$  and fixed helium content of  $Y = 0.23$ . Over all this range of  $Z$ , we find that the ZAHB luminosities determined with models computed by adopting the OPAL EOS are brighter by about 0.05 mag with respect to models computed by means of the Straniero EOS. This is due to a higher efficiency of the hydrogen shell. We checked this last statement by comparing two ZAHB models, one with the OPAL equation of state (except for the very central region  $M_r/M < 0.1$ , not covered by the OPAL tables) and the other with the Straniero (1988) EOS in the whole helium core ( $M_r/M < 0.65$ ) and OPAL EOS elsewhere, in particular in the H-shell region; all the other parameters and physical inputs were kept fixed. We found that the differences in luminosity and effective temperature between the two models are negligible such that the luminosity difference relative to models constructed completely with the old EOS results from the use of the new EOS in the H-shell region.

In Table 1, we summarize isochrone ages compared to CK95. For two metallicities and typical values for  $\Delta(V)$ , we computed CK95 ages (for the OPAL EOS) according to their equation (4) and determined our values from our isochrones and ZAHB models directly. We note that  $[M/H]$  stands for the total metallicity in both papers; although CK95 use the notation  $[Fe/H]$ , their isochrones did include  $\alpha$ -enhancement as well. For the lower metallicity, our method yields an age reduction of 0.5 Gyr; for the higher one, more than 1 Gyr.

To summarize, the introduction of the new EOS is expected to yield an increase in  $\Delta(V)$  of 0.08–0.15 mag, which corresponds to an age reduction of  $\approx 1$  Gyr up to 2 Gyr with the relative contributions from the TO and ZAHB

TABLE 1  
AGE DIFFERENCE BETWEEN THIS WORK AND CHABOYER &  
KIM (1995) FOR FIXED METALLICITY AND  $\Delta(V)$

[M/H]	$\Delta(V)$	Age: CK95 (Gyr)	Age: This Work (Gyr)
-2.00.....	3.45	14.0	13.6
	3.50	14.8	14.3
	3.55	15.5	15.0
-1.70.....	3.45	13.8	12.7
	3.50	14.5	13.3
	3.55	15.3	14.0

depending on the absolute value of the age; in § 3.1, we will determine the age of M68 with the two sets of EOS tables to quantify the effect for one example. Relative to CK95, we expect a reduction between a few  $10^8$  and 1 Gyr.

As a comment, we add that we confirm a result obtained by Salaris, Chieffi, & Straniero (1993): comparing the  $\alpha$ -enhanced isochrones and ZAHBs with the scaled solar ones for the same total metallicity, we find that the  $\alpha$ -enhanced models are well reproduced by scaled solar ones. In contrast to Salaris et al. (1993), we had access to opacity tables reflecting the  $\alpha$ -enhancement for all temperatures. Note that this equivalence holds for low metallicities only (Weiss, Peletier, & Matteucci 1994).

### 2.2. Color Transformations

The color transformations and bolometric corrections used to transform theoretical temperatures and luminosities into magnitudes in the  $UBV$  system come from Buser & Kurucz (1978, 1992, hereafter BK78 and BK92). We adopted the BK92 transformations for  $T \leq 6000$  K and the BK78 at higher temperatures (up to  $T = 10,000$  K) according to the suggestions of BK92. For each metallicity, the two sets have two temperatures in common (5500 and 6000 K, respectively), and we have shifted the  $B - V$  values of BK92 in order to match the corresponding ones of BK78 at these two temperatures. This set of transformations, as discussed by BK92, is highly homogenous and covers all the evolutionary stages displayed in the color-magnitude diagrams (CMDs) of the studied clusters.

Kurucz (1992) provided another set of transformations derived from model atmosphere calculations performed by using updated input physics, covering homogeneously the complete range of effective temperatures and gravities from the MS to the HB evolutionary phase and beyond. There is some concern about the Kurucz ( $B - V$ )- $T_{\text{eff}}$  relation, especially for metal-poor RGB stars (see, e.g., Bell, Paltoglou, & Tripicco 1994; McQuitty et al. 1994; Gratton, Carretta, & Castelli 1996; D'Antona & Mazzitelli 1996; Alexander et al. 1996), but nevertheless we have considered—as a test for the influence of the color transformations and bolometric corrections on the age determination of the clusters—the difference in visual magnitude between the observational ZAHB in the region of the RR Lyrae stars and the TO of the cluster, obtaining an age reduced by  $\approx 0.5$  Gyr with respect to the value obtained adopting the BK78 + BK92 transformations (see the next section). Note that this reduction is well below the other uncertainties in the age determination of the clusters.

### 2.3. HB Luminosity and Distance Modulus

As has already been remarked, we determine absolute cluster ages by comparing the theoretical with the observed

$\Delta(V)$ . We have therefore used theoretical ZAHB models for fixing the moduli of the clusters' distance. This is the main difference between our approach and that of CK95 and Chaboyer et al. (1996) who obtain the distance by comparing an empirical relation for the absolute mean magnitude of RR Lyrae stars as a function of cluster metallicity with the apparent luminosity of the cluster RR Lyrae stars. The distance modulus is a crucial quantity for the cluster ages; it is well known (see, e.g., Salaris et al. 1993) that a variation of 0.1 mag in the distance modulus results in an age variation of  $\approx 1.2$ –1.7 Gyr (where the exact value depends on the absolute age value). It is therefore necessary to discuss the uncertainties related to both methods; we begin with the empirical HB luminosity determination.

The absolute magnitude of RR Lyrae stars is represented by empirical relations of the form

$$M_v(\text{RR}) = a[\text{Fe}/\text{H}] + b. \quad (1)$$

As was shown in several papers (see, e.g., Carney, Storm, & Jones 1992; Chaboyer 1995; Chaboyer et al. 1996), there are several independent observational methods to determine the absolute magnitude of RR Lyrae stars. Depending on the method, the slope  $a$  can vary from 0.08 (Ajhar et al. 1996) to 0.30 and the zero point  $b$  from 1.06 (Layden, Hanson, & Hawley 1994 as quoted by Chaboyer et al. 1996) to 0.73 (Walker 1992). Even if the extremes do not seem to be acceptable (see, e.g., Carney et al. 1992; Caputo & De Santis 1992; Bono et al. 1995) and the discrepancy between the zero points could be reduced to  $\approx 0.15$  (Chaboyer 1995; Chaboyer et al. 1996), these differences in the coefficients still result in very different cluster ages. This is clearly shown in the paper of Chaboyer et al. (1996), where for each considered cluster, they compare the age obtained using different coefficients for equation (1); for M68, for example, their ages range from 12.9 to 17.3 Gyr. Chaboyer et al. (1996) decided to adopt  $M_v(\text{RR}) = 0.20[\text{Fe}/\text{H}] + 0.98$  and ignored the uncertainty in  $a$  and  $b$ ; CK95 took 0.94 for the zero point. This choice already fixes the range of ages one can obtain from the theoretical TO brightness.

A second source of error has recently been mentioned by Castellani, Brocato, & Piersimoni (1996). Using both theoretical ZAHB and TO luminosities at the same time, the choice of available color transformations does not influence  $\Delta(V)$  significantly. This is not the case, however, if one adopts an empirical evaluation of the HB luminosity to determine the distance modulus. Castellani et al. (1996) noted that between the Buser & Kurucz (1978, 1992) and the Kurucz (1992) transformations, there is a difference in the TO magnitude of  $\approx 0.1$  mag, which, if the HB brightness is taken from an empirical RR Lyrae luminosity relation, results in a difference in age of  $\approx 2$  Gyr; we checked and we confirm their results, adding that if we compare the Kurucz (1992) with the Vandenberg & Bell (1985) transformations, the difference in the TO luminosity is almost twice as large ( $\approx 0.18$  mag). While  $\Delta(V)$  is preserved in the case of theoretical HB luminosities, we find that the shape of the isochrones in the TO region changes for the Kurucz (1992) transformations, such that an age determined by a global fit to the CMD might still be influenced.

Regarding the method that we used, the uncertainties related to the theoretical models of horizontal-branch stars are of concern. It is clear that we do not understand HB morphologies in detail. However, this concerns mainly the distribution of stars along the horizontal direction. For the

$\Delta(V)$  method, the luminosity, and in our case that of the ZAHB, is the important quantity, and this seems to be much better defined. HB luminosities depend primarily on  $M_{\text{He}}$ , which is a very stable quantity—when considering canonical evolutionary models—that agrees very well between various investigations (Renzini & Fusi Pecci 1988; Sweigart 1994) in spite of different numerical methods and physical input data. In particular, Straniero & Chieffi (1991) have used the same evolutionary code that we have employed in this investigation, and their results for  $M_{\text{He}}$  are in good agreement with independent computations by Sweigart & Gross (1978) and Vandenberg (1992). The only very discrepant result in the literature is that of Mazzitelli (1989; see also Mazzitelli, D'Antona, & Caloi 1995) who find sensibly larger values by  $\approx 0.01 M_{\odot}$ , which, however, are questioned by, e.g., Sweigart (1994).

The mass of the He core could be different in the presence of noncanonical effects such as fast core rotation. However, this seems to be excluded by synthetic HB models compared to the observations (see, e.g., Caputo et al. 1993; Caputo & De Santis 1992; Lee, Demarque, & Zinn 1990).

HB luminosities also depend on the envelope helium content. Although there is undoubtedly an additional mixing mechanism operating in globular cluster red giants that leads to observed CNO-isotope anomalies, the additional mixing does not change the helium content because it does not extend deep enough into the hydrogen shell (Denissenkov & Weiss 1996). Possible uncertainties in the treatment of (semi)convection of the convective He core do not influence ZAHB model luminosities or temperatures, but only HB lifetimes.

To determine the age of a cluster using the  $\Delta(V)$  method with theoretical ZAHB luminosities, we need to determine the observational equivalent. If the mean luminosity of the cluster's RR Lyrae stars is provided, the observational ZAHB brightness at the RR Lyrae instability strip has been determined by adopting relation (4) from Carney et al. (1992). This relation (also used by Chaboyer et al. 1996) is based on the data taken from the analysis by Sandage (1990) of the vertical extent of the observed and well-populated HBs of a sample of globular clusters, for which the ZAHB level could be unambiguously determined as the sharp lower envelope of the star distribution. It is an empirical relation that provides the ZAHB visual magnitude at the RR Lyrae region as a function of  $[\text{Fe}/\text{H}]$  and of the RR Lyrae average magnitude ( $\langle V_{\text{RR}} \rangle$ ), and it is given by

$$V_{\text{ZAHB}} = \langle V_{\text{RR}} \rangle + 0.05[\text{Fe}/\text{H}] + 0.20 \quad (2)$$

and will be used in the following section.

A possible test for the reliability of our ZAHB models is a direct comparison with empirical  $M_v(\text{RR}) - [\text{Fe}/\text{H}]$  relations. From our models, we derive

$$M_v(\text{ZAHB}) = 0.21[\text{Fe}/\text{H}] + 0.91 \quad (3)$$

when taking into account  $[\alpha/\text{Fe}] \approx 0.3$ . This is in very good agreement with Walker (1992), whose result for the mean RR Lyrae luminosity, after using equation (2), translates into  $M_v(\text{ZAHB}) = 0.20[\text{Fe}/\text{H}] + 0.93$ . A second test is shown in Figure 1, where we compare one of the most recent observational results concerning the absolute luminosity of field and globular cluster RR Lyrae stars versus metallicity (Clementini et al. 1995) and our theoretical predictions for the ZAHB luminosity in the RR Lyrae region. The Clementini et al. results are based on new spectro-

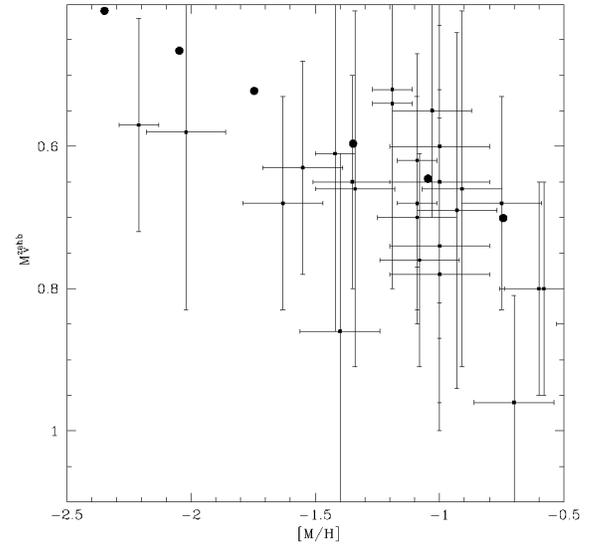


FIG. 1.—Absolute luminosity of supposedly unevolved field and cluster RR Lyrae stars (Clementini et al. 1995) compared to our theoretically predicted ZAHB luminosities (*large filled circles*) for various metallicities. The data are for a conversion factor  $p = 1.38$  between observed and true pulsation velocity (see Fernley 1994 for details).

scopical determinations of metallicity and on Baade-Wesselink estimates of the absolute magnitude of the variables. In Figure 1, only observational data for supposedly unevolved RR Lyrae stars have been considered; these data provide a good estimate for the observed ZAHB luminosities in the RR Lyrae region (G. Clementini 1996, private communication). We have derived the global metallicity  $[M/H]$  for the stars in the sample of Clementini et al. by using the  $[\text{Fe}/\text{H}]$  and  $[\alpha/\text{Fe}]$  values given in their paper; the errors of the observational determination of metallicity and luminosity are also from the quoted paper. The data in the figure refer to one particular prescription for the conversion factor  $p$  (see Fernley 1994) between observed and true pulsation velocities, which is needed for an estimate of the absolute luminosity of the RR Lyrae sample (see Clementini et al. 1995 for more details and their Table 21, col. [6], for the data for  $p = 1.38$ ). One sees that our theoretical models are compatible with the observational data, although there is some offset between our relation and that of Clementini et al. To quantify the comparison, the relation of Clementini et al. (1995) is

$$M_v(\text{RR}) = (0.19 \pm 0.03)[\text{Fe}/\text{H}] + (0.96 \pm 0.04) \quad (4)$$

(eq. [10b] of Clementini et al.), if only the most evolved objects of their sample are excluded. Slope and zero points become, respectively 0.17 and 0.91 (same errors; eq. [10d]), if all possibly evolved objects are excluded as well.

We thus have shown that our theoretical ZAHB luminosities compare well to the results of Walker (1992) and Clementini et al. (1995). This makes them as trustworthy as any of the empirical relations. It also implies that our derived cluster ages could prove to be wrong once a definite answer to the problem of the  $M_v(\text{RR}) - [\text{Fe}/\text{H}]$  has been found.

### 3. CLUSTER FITS

The  $\alpha$ -enhanced isochrones described in the previous section have been used for determining the ages of metal-

poor Galactic globular clusters. We have selected M68 (NGC 4590), M15 (NGC 7078), and M92 (NGC 6341) as representative of the old metal-poor globular cluster population. These three clusters have been extensively studied in the past, and according to various authors (see, e.g., Vandenberg, Bolte, & Stetson 1990; Straniero & Chieffi 1991; Chaboyer et al. 1992; Salaris et al. 1993), they are among the oldest clusters in the Galaxy; therefore, their age places strong constraints on the age of the universe and on the Hubble constant.

Their metallicities are generally considered to be very similar (see, e.g., Hesser & Shawl 1985; Vandenberg et al. 1990), with an average value of  $[\text{Fe}/\text{H}] \approx -2.15$ . If we consider an  $\alpha$  enhancement of  $[\alpha/\text{Fe}] \approx 0.3$  as observed in globular cluster stars, the global metallicity will be  $Z \approx 0.0002$ .

The ages derived in the past years, adopting different theoretical isochrones and different age indicators [such as the  $\Delta(V)$  and the  $\Delta(B-V)$ ; see the discussion in Salaris et al. 1993], range between 15 and 20 Gyr for M15 and M92 and between 13 and 19 Gyr for M68 (see, e.g., Straniero & Chieffi 1991; Carney et al. 1992; Chaboyer et al. 1992). Very recently, by using the OPAL EOS in computing the theoretical evolutionary models, Mazzitelli et al. (1995; but see also Canuto et al. 1996 for an erratum) found an age of around 13–14 Gyr for M68, while CK95 obtained, respectively, 15.9, 14.1, and 12.4 Gyr for M92, M15, and M68. All these authors have used the  $\Delta(V)$  age indicator in order to derive the ages. When using the  $\Delta(B-V)$  method to derive the relative ages of these three clusters, they appear to be coeval, while age differences (as found by CK95) are obtained when using the  $\Delta(V)$  technique. The reason for this discrepancy will be discussed below; in our analysis, the  $\Delta(V)$  method will be used in order to determine the absolute ages of this group of clusters by applying it to the cluster with the best defined CMD (M68); the relative ages between M15, M92, and M68 will be derived using the  $\Delta(B-V)$  technique. To perform this analysis, we have adopted the recent CCD photometries published by Walker (1994) and by Durrell & Harris (1993) for M68 and M15, respectively; in the case of M92, the photometry by Stetson & Harris (1988) has been considered.

### 3.1. M68

We have determined the age of M68 by using our  $\alpha$ -enhanced isochrones for  $Y = 0.23$  and  $Z = 0.0002$ . From the Walker (1994) data, we have derived a ridge line for MS, subgiant branch (SGB), and RGB that is almost coincident with the fiducial line provided by McClure et al. (1987). This is not surprising, since Walker also noted that the fiducial line provided by McClure et al. constitutes an excellent fit to his CMD.

The distance modulus and the cluster reddening have then been determined by shifting the ZAHB models and the isochrones corresponding to ages between 11 and 15 Gyr horizontally and vertically in order to match simultaneously the main-sequence (MS) ridge line and the observational ZAHB level at the RR Lyrae region. The horizontal shift applied in order to match the MS corresponds to the reddening  $E(B-V)$ , while the vertical shift necessary for fitting the observational ZAHB provides the apparent distance modulus  $(m-M)_V$ .

In M68, the RR Lyrae stars are a substantial fraction of the HB population ( $N_{\text{RR}}/N_{\text{HB}} \approx 0.20$ , where  $N_{\text{RR}}$  is the

number of RR Lyrae stars and  $N_{\text{HB}}$  is the total number of HB stars), and it is statistically not possible to assume that all the RR Lyrae stars are evolved off the ZAHB (for a discussion, see Renzini & Fusi Pecci 1988); therefore, the sharp lower envelope of their distribution has to be a reliable estimate of the ZAHB level. For this reason, we have applied equation (2) in order to locate correctly the observational ZAHB at the RR Lyrae region ( $B-V \approx 0.3-0.5$ ). An average RR Lyrae star's magnitude  $\langle V_{\text{RR}} \rangle = 15.64 \pm 0.01$  (as provided by Walker) corresponds to  $V_{\text{ZAHB}} = 15.73$ . An apparent distance modulus  $(m-M)_V = 15.26 \pm 0.10$  is derived from our models, where the error comes from the error in  $\langle V_{\text{RR}} \rangle$  and from the error introduced by the use of equation (2), which is a fit to observational points with a dispersion of around 0.1 mag (see Fig. 4 in Carney et al. 1992). By adopting this distance modulus, the lower envelope of the observed HB is well reproduced by our ZAHB models (see Fig. 2). The reddening derived by this procedure is  $E(B-V) = 0.06$ , in good agreement with  $0.07 \pm 0.01$  given by Walker.

With our derived distance modulus, the observed TO luminosity ( $V_{\text{TO}} = 19.05 \pm 0.05$ ; Walker 1994) is reproduced by our theoretical isochrones for an age of 12.2 Gyr, with a formal error of around  $\pm 1.8$  Gyr (obtained by combining the uncertainties on the TO and ZAHB luminosities). This age is only 0.2 Gyr lower than that of CK95 (OPAL), but it is in agreement with the expectations based on Table 1. To illustrate the fit, the CMD of M68 is displayed in Figure 2, together with the theoretical ZAHB and three isochrones corresponding to different ages (11, 12, and 13 Gyr), shifted according to the derived distance modulus and reddening.

We have also tested the influence of a slightly higher original helium content ( $Y = 0.24$ ) and of an increase of the metallicity by a factor of 2 ( $Z = 0.0004$ , the metallicity used for example by CK95 and also suggested by the spectroscopical determination by Gratton & Ortolani 1989). In both cases, we obtained a slight reduction of the age, by about 1 Gyr. However, the isochrones' shape is changed for

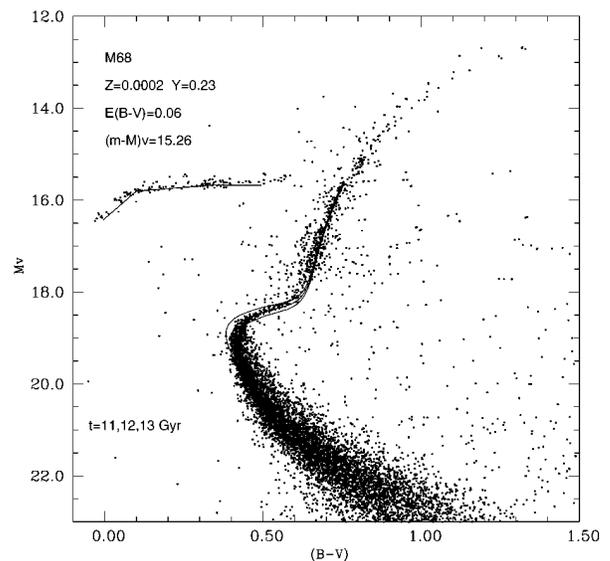


FIG. 2.—Isochrones for ages between 11 and 13 Gyr and ZAHB compared to the CMD of M68 (data from Walker 1994). Composition, distance modulus, and reddening used for the fit are given in the upper left-hand corner.

the higher metallicity such that the fit appears to be worse, especially in the subgiant region.

We note also that if we use for the fit to the CMD of M68 the same set of  $\alpha$ -enhanced isochrones and ZAHB, but the Kurucz (1992) transformations, we obtain a formal age reduction of less than 0.5 Gyr when fitting the observed  $\Delta(V)$ , a reduction well within the observational uncertainties.

Before closing the discussion about M68, we will briefly analyze the influence of the OPAL EOS on the age determination of M68:

1. First of all, we come back to the Straniero (1988) EOS for our isochrones and ZAHB models, while all the other physical inputs are as before. In this case, the luminosity of the ZAHB level in the RR Lyrae region is  $\approx 0.05$  mag lower than in OPAL EOS case and the TO is, for a fixed age of 12 Gyr,  $\approx 0.09$  mag more luminous than in the previous case. The two effects together increase the age by about 1.4 Gyr; by fitting the cluster with the same method as before, we find an age of 13.6 Gyr. This is again in close agreement with the “standard age” of CK95 (13.3 Gyr), but the net age reduction is higher in our case.

2. To separate the effect of our new value for the ZAHB luminosity from that of the updated luminosity at the TO of our isochrones, we fitted the cluster again using the ZAHB level calculated with the Straniero (1988) EOS and the isochrones with the OPAL EOS, obtaining an age of 13.2 Gyr.

### 3.2. M15

In principle, it is possible to determine the absolute age of M15 by adopting the same procedure as for M68 because also in the case of M15, we have a homogenous CCD diagram from the MS to the HB evolutionary phases; the RR Lyrae stars are again a substantial fraction of the HB population ( $N_{RR}/N_{HB} \cong 0.25$ ), and the lower envelope of their distribution therefore has to match the observational ZAHB level. If we apply equation (2) in order to locate correctly the observational ZAHB at the RR Lyrae region,  $\langle V_{RR} \rangle = 15.83 \pm 0.01$  (as provided by Durrell & Harris 1993) corresponds to  $V_{ZAHB} = 15.92$ . When using our  $\alpha$ -enhanced isochrones for  $Y = 0.23$  and  $Z = 0.0002$ , an apparent distance modulus  $(m - M)_V = 15.45 \pm 0.10$  and a reddening of  $E(B - V) = 0.11$  are derived.

The distance modulus and the reddening obtained with our procedure are in good agreement with the results by Durrell & Harris (1993), who found  $(m - M)_V = 15.38 \pm 0.15$  from fitting the local subdwarf sequence to the cluster main sequence and  $E(B - V) = 0.10 \pm 0.01$ . With this distance modulus, the observed TO luminosity ( $V_{TO} \cong 19.4$  according to Durrell & Harris) is reproduced by our theoretical isochrones for an age of 14.5 Gyr. The problem is that the exact value of the TO magnitude and the associated error are not given by Durrell & Harris (1993). In their paper, they state that the TO is around  $V = 19.4$ ; from the fiducial smoothed line presented in their Table 2 (corrected for the typographical error noticed by Walker 1994), it is easy to derive that between  $V = 19.217$  and  $V = 19.420$ ,  $B - V$  varies by only 0.001 mag, which is well inside the error associated with the determination of the fiducial line. Therefore, the exact TO luminosity could well be anywhere between  $V \approx 19.2$  and  $V \approx 19.4$ . This uncertainty of 0.2 mag in the TO magnitude corresponds to an age variation

of approximately 3 Gyr for ages around 14 Gyr. It is therefore evident that the age of 14.5 Gyr previously derived on the basis that  $V_{TO} = 19.4$  constitutes only an upper limit to the cluster age. This age appears to be larger than that of CK95 (14.1 Gyr). However, as Chaboyer et al. (1996) have stated, Chaboyer et al. (1992) had confused observationally determined HB with ZAHB luminosities. CK95 used the same values, too. M15 is one of the clusters affected. Instead of  $\Delta(V) = 3.54$ , they should have used the corrected value of 3.63. Consequently, the correct age would be about 15 Gyr (for the OPAL EOS). Our age—as determined from the formal location of the TO—is therefore somewhat lower owing to the use of the theoretical ZAHB models.

To illustrate the isochrone fit, the CMD of M15 is displayed in Figure 3, together with the theoretical ZAHB and four isochrones corresponding to different ages (between 12 and 15 Gyr), shifted according to the derived distance modulus and reddening. From the figure, it appears that the 14 Gyr isochrone misses the subgiant branch and that the 12 Gyr isochrone would fit better.

For an alternative determination of the age of M15, we rely on the  $\Delta(B - V)$  method (VandenBerg et al. 1990; Stetson et al. 1996) in order to derive the relative age of M15 with respect to M68. As VandenBerg et al. (1990) have shown (but see also Chieffi & Straniero 1989; Salaris, Straniero, & Chieffi 1994), it is possible to derive with high accuracy the relative ages of clusters with approximately the same metallicity (within a difference of about a factor of 2) by considering the difference in  $B - V$  between the TO (whose color is very well defined since the TO region is vertical) and the base of the RGB; in this so-called horizontal method, the color of the lower RGB of two clusters is compared, following a horizontal shift to normalize the TO colors, and a vertical one to force coincidence between the two MS at a position 0.05 mag redder than the TO. Differences in the RGB color (fixed, for example, at a point 2.5 mag more luminous than the MS reference point) then correspond to age differences.

By using this procedure for comparing M68 with M15

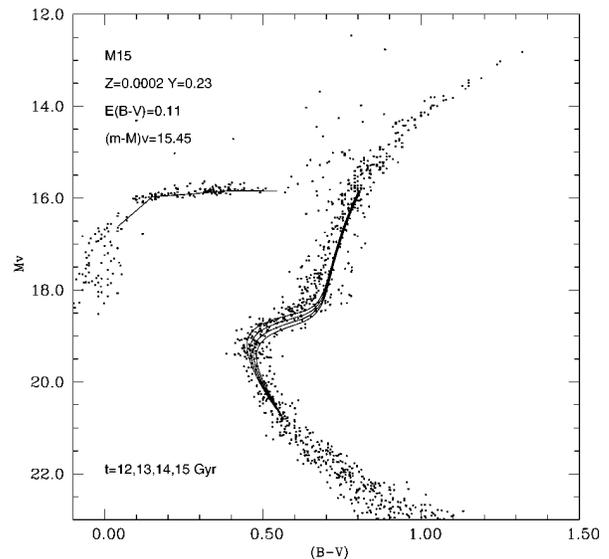


FIG. 3.—Isochrones for different ages (given in Gyr in the figure) fitted to the CMD of M15 (data from Durrell & Harris 1993). Composition, reddening, and distance modulus are displayed in the upper left-hand corner.

(adopting the data of Durrell & Harris 1993), Walker (1994) found that the two RGBs (he used the fiducial line by McClure et al. 1987 for M68 that, as already mentioned, constitutes an excellent fit to his CMD) agree within  $\pm 0.005$  mag in  $B-V$ , which corresponds (adopting our isochrones) to a difference in age of less than  $\pm 0.5$  Gyr. It is worth noting that these relative ages are not modified if the clusters do not have the same metallicity, at least for differences up to a factor of 2 in  $Z$ . (We recall that spectroscopic analyses suggest a higher iron content for M68 as the one we prefer to use; Gratton & Ortolani 1989.)

In Figure 4a, the  $\Delta(B-V)$  of M68 and M15 are compared, following the procedure previously described. The dashed lines that run parallel to the RGB of M68 correspond to an age difference of  $\pm 1$  Gyr with respect to M68, as derived from our theoretical isochrones. The figure clearly shows that the two clusters are virtually coeval, with an absolute age of 12.2 Gyr as derived by means of the  $\Delta(V)$  method applied to M68. Assuming this age for M15 and the distance modulus derived from our ZAHB models, the observational TO luminosity has to be  $V \cong 19.27$ , compatible with the range of values previously discussed.

### 3.3. M92

In the case of M92, no homogenous CCD photometry with well-defined MS, TO, RGB, and a well-populated HB is available; in addition to this, M92 has a very blue HB, and it is not possible to apply the  $\Delta(V)$  method safely in order to determine its absolute age (see, e.g., Vandenberg et al. 1990).

We have considered the photometry of Stetson & Harris (1988), with a very poorly populated HB, and we rely on the analysis by Durrell & Harris (1993) in order to determine the relative age of M92 with respect to M15. By comparing the ridge line provided by Stetson & Harris (1988) with the one obtained for M15, Durrell & Harris derived a difference of 0.009 mag in  $\Delta(B-V)$  between the two clusters that corresponds to an age difference of less than 1 Gyr (with M92 appearing slightly younger). CK95 had determined an

absolute age of about 17 Gyr (15.94 in their paper, corrected again for an erroneous HB luminosity) with the  $\Delta(V)$  method.

In Figure 4b, the  $\Delta(B-V)$  values of M68 and M92 are directly compared. Also in this case, the two dashed lines correspond to an age difference of 1 Gyr with respect to M68. It is evident from the figure that M92 is coeval with or not more than 1 Gyr younger than M68; this confirms that M92, M15, and M68 are extremely uniform in age (as extensively discussed by Vandenberg et al. 1990; Straniero & Chieffi 1991), with an absolute age of about  $12 \pm 2$  Gyr, as determined for M68 by using the  $\Delta(V)$  technique.

## 4. SUMMARY AND DISCUSSION

In this paper, we have reexamined the age of three of the oldest globular clusters (M15, M68, M92). Changes with respect to earlier work included the use of the latest OPAL EOS and opacity tables, supplemented by low-temperature opacity tables for *exactly* the same compositions. In particular,  $\alpha$ -enhancement within the metals has been properly taken into account. For appropriate chemical compositions, both isochrones and ZAHB models were calculated and cluster ages were derived from the  $\Delta(V)$  difference between TO and ZAHB. Our theoretical prediction for the brightness of RR Lyrae stars as a function of metallicity agrees well with that of Walker (1992) and the recent one by Clementini et al. (1995).

CK95 and Mazzitelli et al. (1995) have already used the new OPAL EOS for cluster age determinations. While CK95 show that the improved EOS leads to an age reduction of about 7%, Mazzitelli et al. (1995) argue that the lower ages they obtain result primarily from the use of the new OPAL opacities and their convection theory. In both papers, the  $\Delta(V)$  method has been used as well.

The main difference with CK95 is that we use purely theoretical models to determine  $\Delta(V)$ , while CK95 employed an empirical relation between the absolute mean RR Lyrae brightness and cluster metallicity. Also, the intention of both papers differs: while CK95 investigated the effect of the new EOS on a large sample of clusters with necessarily a wide range of metallicities, ages, and data quality, we intended to determine the age of very old clusters very accurately.

With this approach, we concluded that only M68 can be used with confidence for the  $\Delta(V)$ -method. Our determined age for M68 is  $12.2 \pm 1.8$  Gyr; simultaneously, the fit shown in Figure 2 is very good. While the CMD of M15 is very well suited for the direct age determination, we found that the TO has not been determined accurately enough by the observers. A straightforward determination yields 14.5 Gyr, but we consider the real errors to be as large as 2–3 Gyr. A differential or “horizontal” analysis with respect to M68 shows that both clusters are coeval within less than 0.5 Gyr. An isochrone of this age fits quite well with the TO within the observationally allowed range. Clearly, for this cluster a more accurately determined TO is highly required. Finally, for M92, we used only the horizontal method, because this cluster is not rich enough in red HB stars and no photometric data of sufficient homogeneity are available. Again, M92 results as being coeval with M68 within 1 Gyr. We conclude that the age of the three clusters is about  $12 \pm 2$  Gyr and that all three can be considered as coeval within the uncertainties of age determination. Recently, Richer et al. (1996) came to a similar conclusion; in fact, they con-

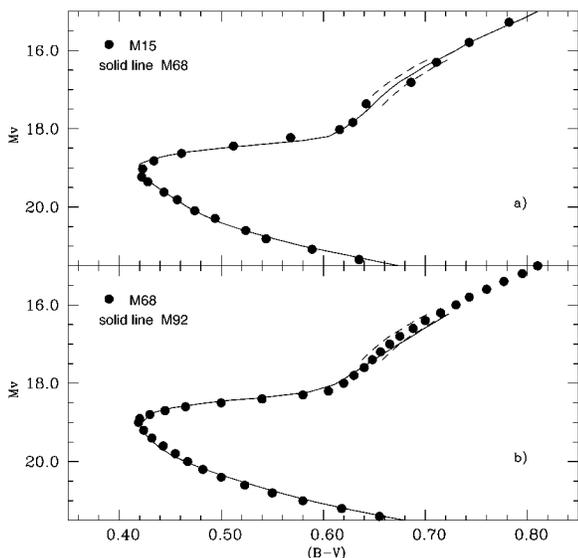


FIG. 4.—Comparison of CMD ridge lines between pairs of clusters, showing them to be coeval. *Upper panel*: M68 (line) and M15 (dots; fiducial line by Durrell & Harris 1993); *lower panel*: M68 (dots) and M92 (line; ridge line by Stetson & Harris 1988). The dashed lines on both sides of the RGBs indicate isochrones of  $\pm 1$  Gyr with respect to the age of M68.

cluded that 11 clusters with  $[Fe/H] < -1.8$  are coeval. We also refer the reader to the independent work of D'Antona, Caloi, & Mazzitelli (1997), who find a similar age for M68 and M30, another metal-poor cluster. This paper also contains an extensive discussion of M92 and the use of subdwarf fitting to obtain the distance modulus.

Compared to CK95, our results differ in several aspects. CK95 find a large age spread of up to 4.5 Gyr (after the error in HB brightness for some cluster has been taken into account) for the three clusters under consideration. This is due to their straightforward use of the  $\Delta(V)$ -method. However, for M68—the cluster with the best data—our derived ages are virtually indistinguishable. Second, the dependence of the isochrone TO brightness on the EOS change appears to be different in detail, although similar in the overall effect. This point needs clarification, but the comparison has to be done with a more detailed grid of metallicities. While they find a rather constant age reduction of 7%, the age reduction in our case depends both on metallicity and age. For M68, it is 10% (relative to the old EOS age) with the additional ZAHB effect amounting to one-third. The TO effect is therefore very similar to CK95 for this cluster.

The derived ages *could further be reduced* by the following means: (1) the use of a higher helium content, justified by big bang nucleosynthesis results on the best-fit predicted primordial helium content of 0.247 (Hata et al. 1996) or by observations including a large systematic error (Olive & Steigman 1995); (2) the inclusion of diffusion (Chaboyer et al. 1992); both factors reduce the ages by another 0.5 Gyr at

least, such that an age of the oldest globular clusters of less than 12 Gyr seems to be in reach; (3) the possibility that our ZAHB fits are too conservative; in fact, inspection of Figures 2 and 3 gives the impression that the true ZAHB might be located even lower, assuming that the ZAHB is the lower envelope of the observed objects. On the other hand, there is no guarantee that our theoretical ZAHB luminosities are the correct ones, although they lie within the range of empirically determined relations. Further efforts and improvements are needed to settle this question, which is of basic importance for any age determination of clusters but also for the extragalactic distance scale and thus for the age of the universe itself.

Interestingly, the mean age obtained for the three clusters is very similar to the age of the disk (10–12 Gyr) obtained by Hernanz et al. (1994) by means of the luminosity function of the white dwarfs in the solar neighborhood; this could imply that the Galactic disk began to form without a substantial time delay with respect to the halo.

In a forthcoming paper, we will present extended results for a large sample of clusters. The bottom line of the present paper is that with updated physics, the best age estimates for the oldest globular clusters indicate that they are only 12 Gyr old. This reduces the “age conflict” drastically.

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