

**Astronomy 25**  
**Useful Equations**  
**Second half of semester**

**Heisenberg Uncertainty principle**

$$\Delta x \Delta p \geq \hbar/2$$

-or-

$$\Delta E \Delta t \geq \hbar/2, \text{ where } \hbar = h/2\pi$$

**Circular velocity**

$$v_c^2 = GM/R$$

**Virial cluster mass**

$$M_{\text{cluster}} = 5Rv_{\text{average}}^2/G$$

**Thermal energy per atom**

$$E_{\text{KE}} \approx kT$$

-where-

$$k = 1.38 \times 10^{-23} \text{ J/K}$$

k = Boltzmann constant

**Einstein's Field Equations**

$$G_{\mu\nu} = (8\pi G/c^4)T_{\mu\nu}$$

-or-

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + g_{\mu\nu}\Lambda = (8\pi G/c^4)T_{\mu\nu}$$

**Friedmann Equation**

$$(8\pi G/3)\rho = H^2 + K$$

**Concordance cosmology**

$$\rho_{\text{cosmos}} = 0.05\rho_{\text{visible}} + 0.25\rho_{\text{dark matter}} + 0.70\rho_{\text{dark energy}}$$

-or-

$$\rho = 0.05\rho_{\text{crit}} + 0.25\rho_{\text{crit}} + 0.70\rho_{\text{crit}} = \rho_{\text{crit}}$$

**Stress-energy tensor with Dark Energy**

$$T_{\mu\nu} = \text{energy density} + 3 \times (\text{pressure})$$

**Matter & energy densities decrease with size scale**

$$\rho_{\text{matter}} = \rho/R^3 \quad (\text{applies to visible/dark matter})$$

$$\rho_{\text{energy}} = \rho/R^4 \quad (\text{applies to non-dark energy})$$

$$\rho_{\text{energy}} = \text{constant} \quad (\text{applies to dark energy})$$

**Black hole parameters**

$$(M_{\odot} = 1 \text{ solar mass})$$

*Schwarzschild radius (again)*

$$R_s = 2.96 \times (M/M_{\odot}) \text{ km}$$

*Effective temperature*

$$T_{\text{BH}} = 6 \times 10^{-8} / (M/M_{\odot}) \text{ K}$$

*Power output*

$$P_{\text{BH}} = 9 \times 10^{-29} / (M/M_{\odot})^2 \text{ W}$$

*Evaporation time*

$$t_{\text{BH}} = 2.1 \times 10^{67} \times (M/M_{\odot})^3 \text{ years}$$

*Tidal forces*

$$\Delta g = 2GMh/r^3$$